AP Calculus BC Summer Assignment

This packet is a review of some Pre-Calculus topics and some Calculus topics. It is to be completed neatly and on a separate sheet of paper. Only use a calculator if absolutely necessary. Credit will be awarded if and only if the correct work is shown, and that work leads to the correct solution. This completed packet is due by the first day of class. If you have any questions you cannot resolve on your own, you may email me at marrante@oxfordasd.org. Have a great summer and I am looking forward to seeing you in late August.

Part I: Let's wet your appetite with a little Precalc!

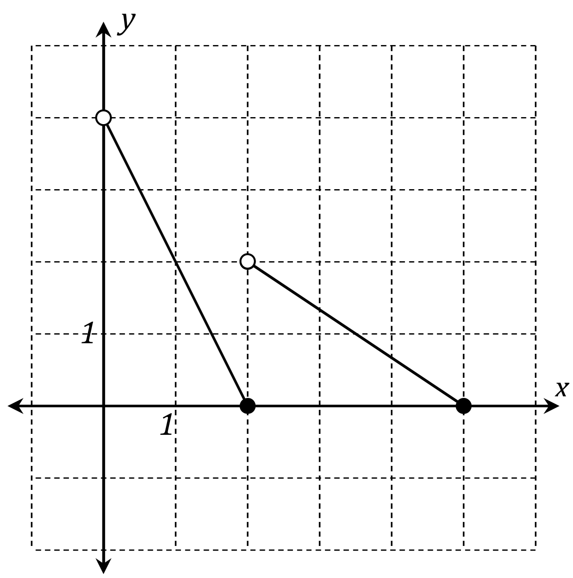
1. For what value of k are the two lines $2x + ky = 3$ and $x + y = 1$
   a. Parallel?
   b. Perpendicular?

2. Consider the circle of radius 5 centered at (0, 0). Find an equation of the line tangent to the circle at the point (3, 4) in slope-intercept form.

3. Graph the piecewise function shown below.

$$f(x) = \begin{cases} 4 - x^2, & x < 1 \\ \frac{3}{2}x + \frac{3}{2}, & 1 \le x \le 3 \\ x + 3, & x > 3 \end{cases}$$

4. Write a piecewise equation for the function shown below. Include the domain of each piece.



5. Graph the function $y = 3e^{-x} - 2$ and indicate asymptote(s). State its domain, range, and intercepts.

Part II: Unlimited and Continuous!

6. For each problem below, find the limits, if they exist.

   a. $\lim\limits_{x \to 4} \dfrac{2x^3 - 7x^2 - 4x}{x - 4}$ b. $\lim\limits_{x \to 9} \dfrac{\sqrt{x} - 3}{9 - x}$

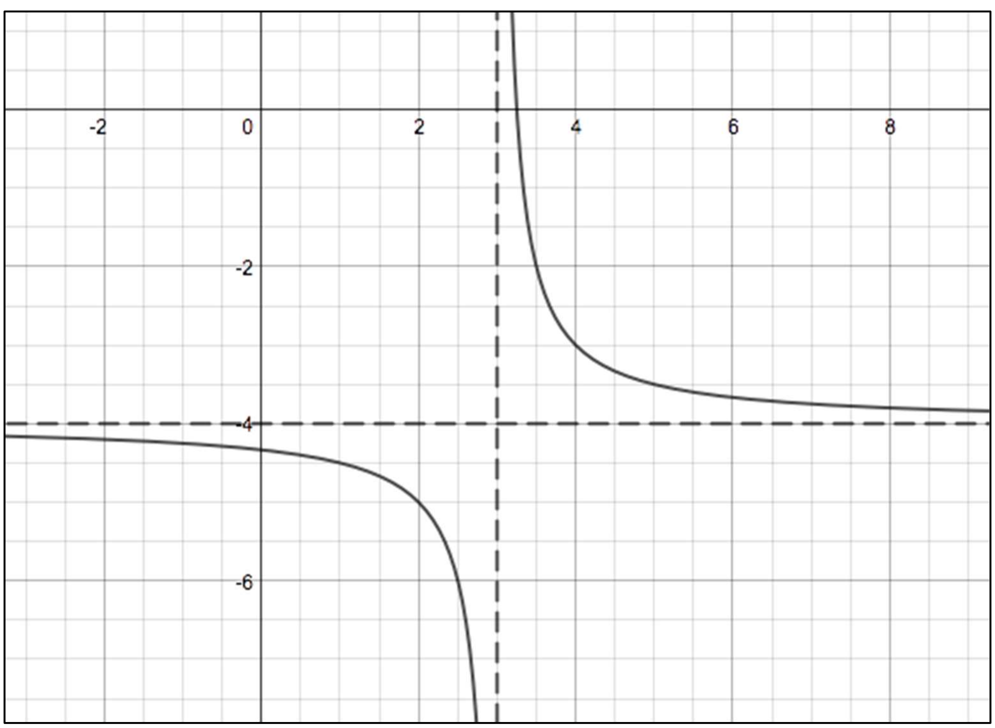
   c. $\lim\limits_{x \to 1} \dfrac{x^2 - 2x - 5}{x + 1}$ d. $\lim\limits_{x \to -2} \dfrac{x^3 + 8}{x + 2}$

7. Explain why each function below is discontinuous and determine if the discontinuity is removable or non-removable.

   a. $g(x) = \begin{cases} 2x - 3, & x < 3 \\ -x + 5, & x \geq 3 \end{cases}$

   b. $b(x) = \dfrac{x(3x+1)}{3x^2 - 5x - 2}$ c. $h(x) = \dfrac{\sqrt{x^2 - 10x + 25}}{x - 5}$

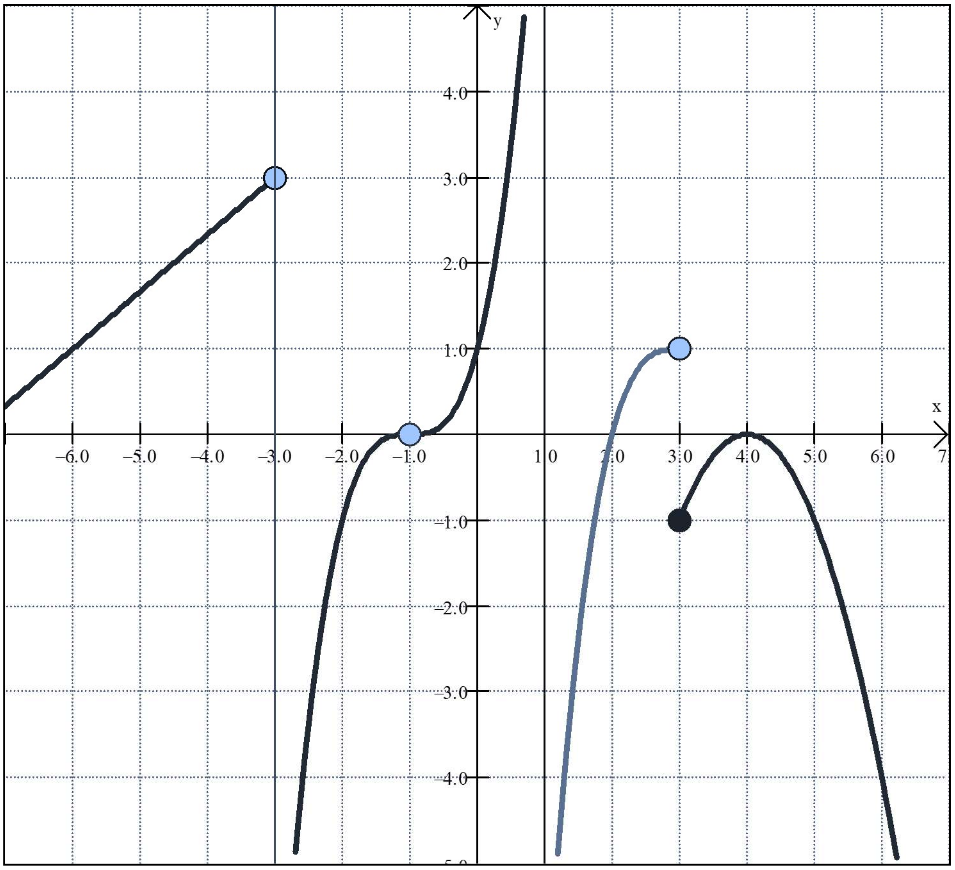
8. Use the graph of $f(x)$ shown below to complete the following problems. (Asymptotes are indicated with dashed lines.)



   a. For what value of a is $\lim\limits_{x \to a} f(x)$ nonexistent?

   b. $\lim\limits_{x \to \infty} f(x) =$ c. $\lim\limits_{x \to -\infty} f(x) =$

9. Determine if the following limits exist, based on the graph below of $p(x)$. If the limit exists, state its value. (Note that x = -3 and x = 1 are vertical asymptotes.)



a. $\lim_{x \to 1^-} p(x)$

b. $\lim_{x \to 3^-} p(x)$

c. $\lim_{x \to 2} p(x)$

d. $\lim_{x \to 3} p(x)$

e. $\lim_{x \to 3^+} p(x)$

f. $\lim_{x \to -1} p(x)$

10. Consider the function $f(x) = \begin{cases} x^2 + kx, & x \le 5 \\ 5\sin\left(\dfrac{\pi}{2}x\right), & x > 5 \end{cases}$,

In order for the function to be continuous at $x = 5$, what must be the value of k?

11. Consider the function $f(x) = \begin{cases} \dfrac{\sin x}{x}, & x \neq 0 \\ k, & x = 0 \end{cases}$

In order for the function to be continuous at $x = 0$, what must be the value of k?

Part III: Designated Deriving!

12. Find each derivative.

a. $y = \ln(1+e^x)$

b. $y = \csc(1+\sqrt{x})$

c. $y = \left(\tan^2 x\right)\left(3\pi x - e^{2x}\right)$

d. $y = \sqrt[7]{x^3 - 4x^2}$

e. $f(x) = (x+1)e^{3x}$

f. $f(x) = \dfrac{e^{x/2}}{\sqrt{x}}$

13. Consider the function $f(x) = \sqrt{x-2}$. On what interval(s) are the hypotheses of the Mean Value Theorem satisfied?

14. If $xy^2 - y^3 = x^2 - 5$, then $\dfrac{dy}{dx} =$

15. The distance of a particle from its initial position is given by $s(t) = t - 5 + \dfrac{9}{t+1}$, where s is in feet and t is in minutes. Find the velocity at $t = 1$ minute in appropriate units.

16. Use the table below for the following exercises.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	4	2	5	$\frac{1}{2}$
3	7	-4	$\frac{3}{2}$	-1

a. The value of $\dfrac{d}{dx}(f \cdot g)$ at $x = 3$ is

b. The value of $\dfrac{d}{dx}\left(\dfrac{f}{g}\right)$ at $x = 1$ is

17. Use the table below to find the value of the first derivative of the given functions for the given value of x.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	2	0	$\frac{3}{4}$
2	7	–4	$\frac{1}{3}$	–1

a. $\left[f(x)\right]^2$ at $x = 2$

b. $f(g(x))$ at $x = 1$

18. Let f be the function defined by $f(x) = \dfrac{x + \sin x}{\cos x}$ for $-\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$.

a. State whether f is an even function or an odd function. Justify your answer.

b. Find $f'(x)$.

c. Write an equation for the line tangent to the graph of f at the point $(0, f(0))$.

Part IV: Derived and Applied!

19. For the exercises below, find all critical values, intervals of increasing and decreasing, any local extrema, points of inflection, and all intervals where the graph is concave up and concave down.

a. $f(x) = \dfrac{5 - 4x + 4x^2 - x^3}{x - 2}$

b. $y = 3x^3 - 2x^2 + 6x - 2$

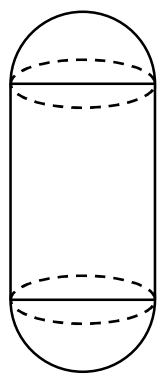
c. $f'(x) = 5x^3 - 15x + 7$

20. At what value(s) of x does the graph of the function $y = x^5 - x^2 + \sin x$ change concavity?

21. Find the equation of the line tangent to the function $y = \sqrt[4]{x^7}$ at $x = 16$.

22. For what value of x is the slope of the tangent line to $y = x^7 + \dfrac{3}{x}$ undefined?

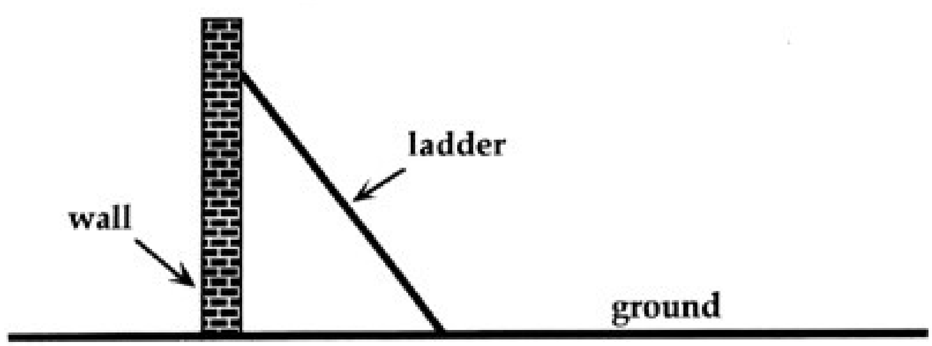
23. A balloon is in the shape of a cylinder with hemispherical ends of the same radius as that of the cylinder. The balloon is being inflated at the rate of 261π cubic centimeters per minute. At the instant the radius of the cylinder is 3 centimeters, the volume of the balloon is 144π cubic centimeters and the radius of the cylinder is increasing at the rate of 2 centimeters per minute. (The volume of a cylinder with radius r and height h is $V = \pi r^2 h$, and the volume of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.)



a. At this instant, what is the height of the cylinder?

b. At this instant, how fast is the height of the cylinder increasing?

24. A ladder 15 feet long is leaning against a building so that one end is on level ground and the other end is on the wall. As shown in the figure. The bottom of the ladder is moving away from the wall at a constant rate of $\frac{1}{2}$ foot per second.



a. Find the rate, in feet per second, at which the top of the ladder is moving when the bottom of the ladder is 9 feet from the wall.

b. Find the rate of change in square feet per second of the area of the triangle formed by the ladder, wall, and ground when the ladder is 9 feet from the building.

Part V: Integral to Your Success!

25. For the exercises below, determine the given integral.

a. $\displaystyle\int_{-8}^{-1} \frac{x-x^2}{2\sqrt[3]{x}}\,dx$

b. $\displaystyle\int_{-\pi/6}^{\pi/6} \sec^2 x\,dx$

c. $\displaystyle\frac{d}{dx}\int_{1}^{x} \sqrt[4]{t}\,dt$

d. $\displaystyle\frac{d}{dx}\int_{\sin(4x)}^{0} e^t\,dt$

e. $\displaystyle\int \frac{x^3}{\sqrt{1+x^4}}\,dx$

f. $\displaystyle\int \frac{\csc^2 x}{\cot^3 x}\,dx$

g. $\displaystyle\int \sqrt{\tan x}\,\sec^2 x\,dx$

26. What are all the values of *k* for which $\displaystyle\int_{2}^{k} x^5\,dx = 0$?

27. What is the average value of $y = x^3\sqrt{x^4+9}$ on the interval [0, 2]?

28. If $\displaystyle\int_{a}^{b} g(x)\,dx = 4a+b$, then $\displaystyle\int_{a}^{b}\left[g(x)+7\right]dx =$

29. The function *f* is continuous on the closed interval [1, 9] and has the values given in the table below. Using the subintervals [1, 3], [3, 6], and [6, 9], what is the value of the trapezoidal approximation of $\displaystyle\int_{1}^{9} f(x)\,dx$?

x	1	3	6	9
f(*x*)	15	25	40	30

30. The table below provides data points for the continuous function $y = h(x)$.

x	0	2	4	6	8	10
$h(x)$	9	25	30	16	25	32

Use a right Riemann sum with 5 subdivisions to approximate the area under the curve of $y = h(x)$ on the interval [0, 10].

31. A particle moves along the x-axis so that, at any time $t \geq 0$, its acceleration is given by $a(t) = 6t + 6$. At time $t = 0$, the velocity of the particle is –9, and its position is –27.

a. Find $v(t)$, the velocity of the particle at any time $t \geq 0$.

b. For what values of $t \geq 0$ is the particle moving to the right?

c. Find $x(t)$, the position of the particle at any time $t \geq 0$.

Part VI: Apply Those Integrals!

32. Find the general solution to each given differential equation.

a. $\dfrac{dy}{dx} = \dfrac{3y}{2+x}$

b. $\dfrac{dy}{dx} = y\sin x$

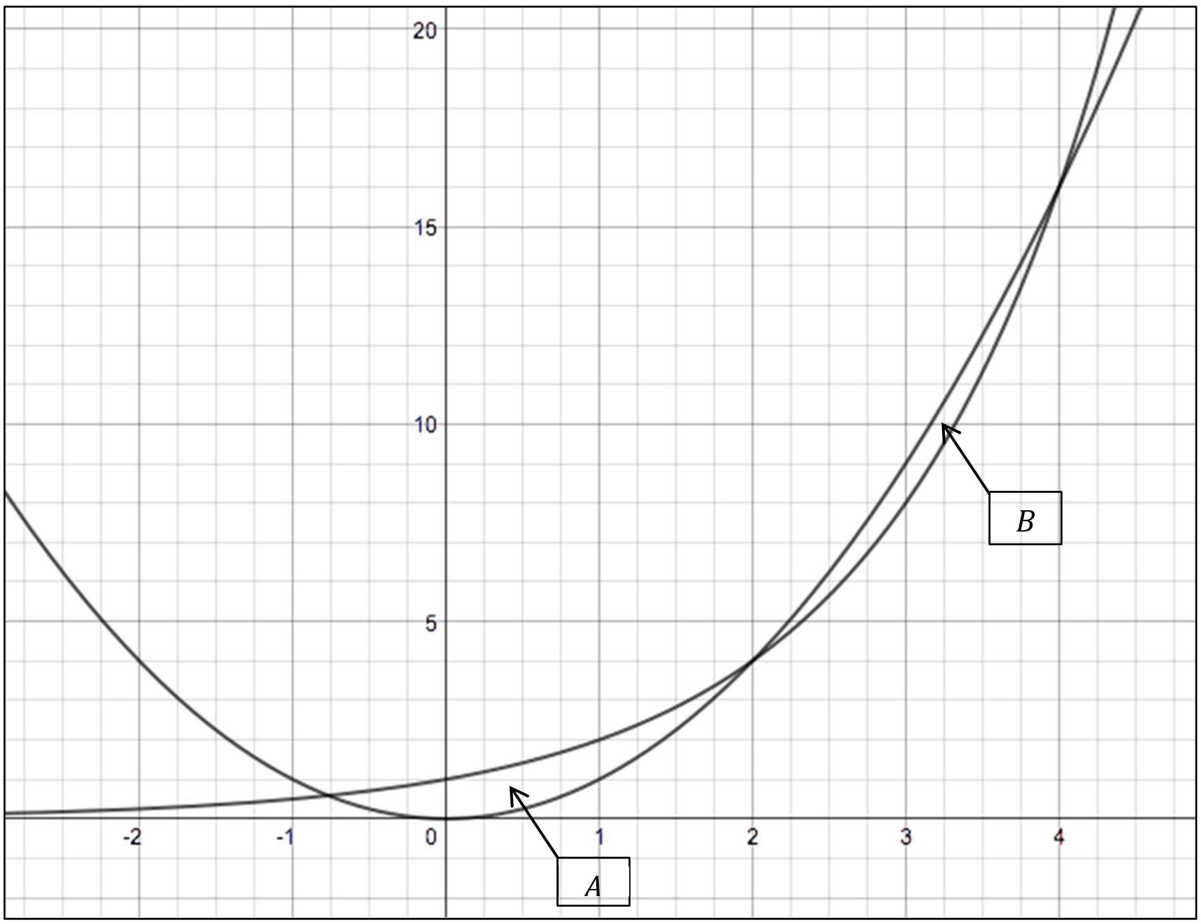
33. Find the particular solution to the differential equation $\dfrac{dy}{dx} = xy\sin x^2$ if $y(0) = 1$.

34. The shaded regions *A* and *B* shown below are enclosed by the graphs of $f(x) = x^2$ and $g(x) = 2^x$

a. Find the *x*- and *y*-coordinates of the three points of intersections of the graphs of *f* and *g*.

b. Without using absolute value, set up an expression involving one or more integrals that gives the total area of regions *A* and *B*. Do not evaluate.

c. Without using absolute value, set up an expression involving one or more integrals that gives the volume of the solid generated by revolving the region *A* about the line $y = 5$. Do not evaluate.



35. Let R be the region in the first quadrant under the graph of $y = \frac{1}{\sqrt{x}}$ for $4 \le x \le 9$.

a. Find the area of R.

b. If the line $x = k$ divides the region R into two regions of equal area, what is the value of k?

c. Find the volume of the solid whose base is the region R and whose cross sections cut by planes perpendicular to the x-axis are squares.